



# Workshop on Integer and Combinatorial Optimization

March 11 – 12, 2020  
in Aachen

Lecture Hall Ho9, C.A.R.L.  
Claßenstraße 11 in 52072 Aachen



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# Program

## Wednesday, March 11

Time	Speaker	Title	Abstract
1:30pm	Registration		
2:00pm	Laura Sanità	On the diameter and the circuit-diameter of a polytope	Page 3
2:50pm	Sarah Morell	Single source unsplittable flows with arc-wise lower and upper bounds	Page 3
3:10pm	Dominik Kamp	Models for determining optimal safety stocks in multi-echelon inventory systems with outsourcing options	Page 3
3:30pm		Coffee Break	
4:00pm	Melanie Schmidt	Approximation Algorithms for Clustering with Side Constraints	Page 4
4:50pm	Tabea Krabs	Minimum Color-Degree Perfect $b$ -Matching Problem	Page 4
5:10pm		Coffee Break	
5:20pm	Jannik Matuschke	Rounding Assignment LPs for Load-Balancing in Scheduling and Beyond	Page 4
6:10pm	Daniele Catanzaro	On the Balanced Minimum Evolution Polytope	Page 4
6:30pm		Break	
8:00pm		Dinner	Page 8

## Thursday, March 12

Time	Speaker	Title	Abstract
9:00am	Sebastian Pokutta	Discrete Optimization in Machine Learning	Page 5
9:50am		Discussion on promising research directions at intersection of Machine Learning and Discrete Optimization	
10:30am		Coffee Break	
11:00am	Vera Traub	Reducing Path TSP to TSP	Page 5
11:50am	Johannes Thürauf	Radius of Robust Feasibility for Mixed-Integer Problems	Page 5
12:10pm	Felix Hommelsheim	Flexible Graph Connectivity: Approximating Network Design Problems Between 1- and 2-connectivity	Page 6
12:30pm		Lunch Break (choose your restaurant)	
2:30pm	Tim Oosterwijk	Flows over Time and Dynamic Equilibria	Page 6
2:50pm	Thomas Rothvoss	Discrepancy Theory: Algorithms and Applications	Page 6
3:40pm	Matthias Walter	Persistency for Linear Programming Relaxations for the Stable Set Problem	Page 7

All talks will be given in the lecture hall Ho9 in the Central Auditorium for Research and Learning (C.A.R.L.), Claßenstraße 11 in 52072 Aachen.

## Abstracts

### On the diameter and the circuit-diameter of a polytope

(Laura Sanità, Eindhoven University)

**Wednesday, 2:00pm** The diameter of a polytope  $P$  is the maximum length of a shortest path between a pair of vertices of  $P$ , when one is allowed to walk on the edges (1-dimensional faces) of  $P$ . Despite decades of studies, it is still not known whether the diameter of a  $d$ -dimensional polytope with  $n$  facets can be bounded by a polynomial function of  $n$  and  $d$ . This is a fundamental open question in discrete mathematics, motivated by the (still unknown) existence of a polynomial pivot rule for the Simplex method for solving Linear Programs. A generalized notion of diameter, recently introduced in the literature, is that of circuit-diameter, defined as the maximum length of a shortest path between two vertices of  $P$ , where the path can use all edge directions (called circuits) that can arise by translating some of the facets of  $P$ . In this talk I will survey some results on the complexity of computing the diameter of a polytope, and the computation of circuits. We will discuss the more general framework of circuit-based augmentation algorithms, and how they can be exploited also to analyze the performance of Simplex method.

### Single source unsplittable flows with arc-wise lower and upper bounds

(Sarah Maria Morell, TU Berlin)

**Wednesday, 2:50pm** In a digraph with a source and several destination nodes with associated demands, an unsplittable flow routes each demand along a single path from the common source to its destination. Given some flow  $x$  that is not necessarily unsplittable but satisfies all demands, it is a natural question to ask for an unsplittable flow  $y$  that does not deviate from  $x$  by too much, i.e.,  $y \sim x$  for all arcs  $a$ . Twenty years ago, in a landmark paper, Dinitz, Garg, and Goemans proved that there is an unsplittable flow  $y$  such that  $y(a)$  is at most  $x(a) + d_{\max}$  for all arcs  $a$ , where  $d_{\max}$  denotes the maximum demand value. Unsplittable flows with arc-wise lower bounds have, to the best of the authors' knowledge, not been considered yet. Based upon an entirely new approach, we prove the following result: There is an unsplittable flow  $y$  such that  $y(a)$  is at least  $x(a) - d_{\max}$  for all arcs  $a$ . Our second contribution is a considerably simpler one-page proof for the above mentioned classical result of Dinitz, Garg, and Goemans, using a variant of our approach. Finally, building upon an iterative rounding technique previously introduced by Kolliopoulos and Stein and Skutella, we prove existence of an unsplittable flow that simultaneously satisfies the upper and lower bounds for the special case when demands are integer multiples of each other. For arbitrary demand values, we prove the slightly weaker simultaneous bounds with  $y(a)$  being lower bounded by  $x(a)/2 - d_{\max}$  and upper bounded by  $2x(a) + d_{\max}$  for all arcs  $a$ .

This is joint work with Martin Skutella.

### Models for determining optimal safety stocks in multi-echelon inventory systems with outsourcing options

(Dominik Kamp, Bayreuth University)

**Wednesday, 3:10pm** In safety stock optimization of multi-echelon supply chains there are mainly two modelling paradigms: the Stochastic Service Model (SSM) and the Guaranteed Service Model (GSM). The GSM approach uses additional optimizable guaranteed service times to derive inventory necessities in order to prevent stock-outs at minimal cost. Therefore it can handle more general demand settings compared to the SSM, which explicitly deals with stock-out induced delays in the supply process. This talk will introduce a project of the German Research Foundation (DFG) in cooperation with Volkswagen (VW) on the problem of incorporating explicit demand propagation into the GSM. This

extension is particularly important in case of the availability of outsourcing options at internal stock points and is currently attacked by techniques of large scale mixed integer programming (MILP).

## **Approximation Algorithms for Clustering with Side Constraints (Melanie Schmidt, Cologne University)**

**Wednesday, 4:00pm** Clustering is a major unsupervised learning tool which has been studied a lot, not only in machine learning but also in the fields of combinatorial optimization and approximation algorithms. There are countless algorithms for various clustering objectives, and many mathematical objective functions like  $k$ -median and  $k$ -center are very well studied. However, the picture changes slightly when clustering objectives appear as constrained problems. Many common algorithms will not provide a way to enforce a side constraint on the solution, and viewed from a theoretical point of view, side constraints are a problem because they often interfere with the best known techniques for obtaining approximate solutions. In this talk, we discuss different clustering problems with side constraints.

## **Minimum Color-Degree Perfect $b$ -Matching Problem (Tabea Krabs, RWTH Aachen)**

**Wednesday, 4:50pm** The minimum color-degree perfect  $b$ -matching problem (ColBM) is a new extension of the perfect  $b$ -matching problem to edge-colored graphs. The objective of ColBM is to minimize the maximum number of differently colored edges in a perfect  $b$ -matching that are incident to the same node. In this talk, we show that ColBM is NP-hard on bipartite graphs by a reduction from 2B-3SAT, and conclude that there exists no  $(2 - \epsilon)$ -approximation algorithm unless  $P = NP$ . However, we identify a class of two-colored complete bipartite graphs on which we can solve ColBM in polynomial time.

## **Rounding Assignment LPs for Load-Balancing in Scheduling and Beyond (Jannik Matuschke, KU Leuven)**

**Wednesday, 5:20pm** A classic result by Lenstra, Shmoys, and Tardos established a 2-approximation for makespan minimization in unrelated machine scheduling by exploiting a sparseness property of extreme point solutions to the natural assignment linear programming relaxation for the problem. We revisit this idea and discuss how it can be applied in more general contexts involving assignment costs and set-dependent load functions. To illustrate this flexibility we review recent results for scheduling splittable or malleable jobs as well as for vehicle routing with depot capacities.

## **On the Balanced Minimum Evolution Polytope (Daniele Catanzaro, Louvain University)**

**Wednesday, 6:10pm** Recent advances on the polyhedral combinatorics of the Balanced Minimum Evolution Problem (BMEP) enabled the characterization of a number of facets of its convex hull (also referred to as the BMEP polytope) as well as the discovery of connections between this polytope and the permutoassociahedron. In this article, we extend these studies, by presenting new results concerning some fundamental characteristics of the BMEP polytope, new facet-defining inequalities in the case of six or more taxa, a number of valid inequalities, and a polynomial time oracle to recognize its vertices. Our aim is to broaden understanding of the polyhedral combinatorics of the BMEP with a view to developing new and more effective exact solution algorithms.

Joint work with Raffaele Pesenti and Laurence Wolsey.

## **Discrete Optimization in Machine Learning (Sebastian Pokutta, TU Berlin and Zuse Institut Berlin)**

**Thursday, 9:00am** Machine Learning problems are often linked to empirical risk minimization problems that in turn are solved with continuous optimization methods. More recently however, it has been recognized that certain problems in machine learning naturally lead to discrete optimization problems. In this talk I will give an overview of the role of discrete optimization methods in machine learning.

## **Reducing Path TSP to TSP (Vera Traub, Bonn University)**

**Thursday, 11:00am** We present a black-box reduction from Path TSP to the classical tour version (TSP). More precisely, we show that given an  $\alpha$ -approximation algorithm for TSP, then, for any  $\varepsilon > 0$ , there is an  $(\alpha + \varepsilon)$ -approximation algorithm for the more general Path TSP. This reduction implies that the approximability of Path TSP is the same as for TSP, up to an arbitrarily small error. This avoids future discrepancies between the best known approximation factors achievable for these two problems, as they have existed until very recently. A well-studied special case of TSP, Graph TSP, asks for tours in unit-weight graphs. Our reduction shows that any  $\alpha$ -approximation algorithm for Graph TSP implies an  $(\alpha + \varepsilon)$ -approximation algorithm for its path version. By applying our reduction to the 1.4-approximation algorithm for Graph TSP by Sebő and Vygen, we obtain a polynomial-time  $(1.4 + \varepsilon)$ -approximation algorithm for Graph Path TSP, improving on a recent 1.497-approximation algorithm of Traub and Vygen. We obtain our results through a variety of new techniques, including a novel way to set up a recursive dynamic program to guess significant parts of an optimal solution. At the core of our dynamic program we deal with instances of a new generalization of (Path) TSP which combines parity constraints with certain connectivity requirements. This problem, which we call  $\Phi$ -TSP, has a constant-factor approximation algorithm and can be reduced to TSP in certain cases when the dynamic program would not make sufficient progress.

This is joint work with Jens Vygen and Rico Zenklusen.

## **Radius of Robust Feasibility for Mixed-Integer Problems (Johannes Thürauf, Nürnberg University)**

**Thursday, 11:50am** For a mixed-integer linear problem (MIP) with uncertain constraints, the radius of robust feasibility (RRF) determines a value for the maximal “size” of the uncertainty set such that robust feasibility of the MIP can be guaranteed. Special cases of the RRF are used to compute a “most robust” solution w.r.t. the size of the uncertainty set in applications such as facility location design. The approaches for the RRF in the literature focus on continuous optimization problems. In this talk, we first analyze relations between the RRF of a MIP and its continuous linear (LP) relaxation. We derive conditions under which a MIP and its LP relaxation have the same RRF. We then extend the RRF such that it can be applied to a large variety of optimization problems and uncertainty sets. In contrast to the setting commonly used in the literature, for every constraint we consider a potentially different uncertainty set that is not necessarily full-dimensional. Thus, we generalize the RRF to MIPs as well as to include “safe” variables and constraints, i.e., where uncertainties do not affect certain variables or constraints. In this setting, we again analyze relations between the RRF for a MIP and its LP relaxation. Afterward, we present methods for computing the RRF of LPs as well as of MIPs with safe variables and constraints. We finally show that the new methodologies can be successfully applied to the instances in the MIPLIB 2017 for computing the RRF.

This is joint work with Frauke Liers and Lars Schewe.

## **Flexible Graph Connectivity: Approximating Network Design Problems Between 1- and 2-connectivity**

**(Felix Hommelsheim, TU Dortmund)**

**Thursday, 12:10pm** Graph connectivity and network design problems are among the most fundamental problems in combinatorial optimization. The minimum spanning tree problem, the two edge-connected spanning subgraph problem (2-ECSS) and the tree augmentation problem (TAP) are all examples of fundamental well-studied network design tasks that postulate different initial states of the network and different assumptions on the reliability of network components. In this paper we motivate and study Flexible Graph Connectivity (FGC), a problem that mixes together both the modeling power and the complexities of all aforementioned problems and more. In a nutshell, FGC asks to design a connected network, while allowing to specify different reliability levels for individual edges. In this paper we develop a general algorithmic approach for approximating FGC that yields approximation algorithms with ratios that are close to the best known bounds for many special cases, such as 2-ECSS and TAP. Our algorithm and analysis combine various techniques including a weight-scaling algorithm, a charging argument that uses a variant of exchange bijections between spanning trees and a factor revealing non-linear optimization problem.

This is joint work with David Adjiashvili and Moritz Mühlenthaler.

## **Flows over Time and Dynamic Equilibria**

**(Tim Oosterwijk, Maastricht University)**

**Thursday, 2:30pm** Dynamic network flows, or network flows over time, constitute an important model for real-world situations where steady states are unusual, such as urban traffic and the Internet. In order to describe the temporal evolution of such systems one has to consider the propagation of flow across the network by tracking the position of each particle along time. These applications immediately raise the issue of analyzing dynamic network flows from a game-theoretic perspective. In this talk I will discuss dynamic equilibria in the deterministic fluid queuing model in single-source single-sink networks, arguably the most basic model for flows over time. In this model, every arc has a capacity and if the inflow into the arc exceeds its capacity, a queue starts to build. After queuing, particles traverse the arc during a fixed transit time. I will show the structure of dynamic equilibria in this model and discuss what is known, and some intriguing open questions.

This is based on a paper that bounds the Price of Anarchy in this model, which is joint work with José Correa and Andrés Cristi.

## **Discrepancy Theory: Algorithms and Applications**

**(Thomas Rothvoss, Washington University)**

**Thursday, 2:50pm** An exciting line of work, starting with Bansal's seminal paper provides algorithms to find partial colorings in polytopes and more generally in large enough convex sets. The original motivation comes from discrepancy theory where the goal is to bi-color a given set of sets as evenly as possible. For example, this enables a constructive solution to Spencer's "6-standard deviations suffice" Theorem. We show a surprising simple deterministic algorithm to find partial colorings in a polytope. Moreover we show how discrepancy theory can be used to obtain linear size spectral sparsifiers in a graph, matching the result of Batson, Spielman and Srivastava. In particular such sparsifiers imply that in any undirected graph one can select a linear size set of edges with weights so that every cut is preserved up to a  $(1 + \epsilon)$  factor.

This is joint work with Avi Levy, Harish Ramadas, and Victor Reis.

## **Persistency for Linear Programming Relaxations for the Stable Set Problem (Matthias Walter, Twente University)**

**Thursday, 3:40pm** The Nemhauser-Trotter theorem states that the standard LP formulation for the stable set problem has a remarkable property, also known as (weak) persistency: for every optimal LP solution that assigns integer values to some variables, there exists an optimal integer solution in which these variables retain the same values. While the standard LP is defined by only non-negativity and edge constraints, a variety of stronger LP formulations have been studied and one may wonder whether any of them has the this property as well. We show that any stronger LP formulation that satisfies mild conditions cannot have the persistency property on all graphs, unless it is always equal to the stable-set polytope.

# Dinner

The dinner starts at 8.00pm. Please indicate at the registration desk if you plan to join us. We reserved tables at the restaurant *Kaiserwetter*.

Kaiserwetter Karree  
(Inside Kapuziner Karree)  
Alter Posthof 13  
52062 Aachen

Google Maps Link: <https://goo.gl/maps/PL22kGqeKcczAEZp7>

There will be a joint walk to the restaurant from the *Karlsbrunnen* in front of Aachen City Hall starting at 7:40pm. Yes, you will see the cathedral on the way.

